1970
USDA Forest Service Research Paper PNW-110

A Technique for the Solution of Skyline Catenary Equations

Ward W. Carson
and
Charles N. Mann

Pacific Northwest Forest and Range Experiment Station
US Department of Agriculture
Forest Service
Portland, Oregon

NOMENCLATURE

(A)	Left support point (see fig. 3).
(B)	Right support point (see fig. 3).
(C)	Carriage location (see fig. 3).
d	Horizontal distance between support point (A) and the carriage position (C).
e_i	Length of horizontal moment arm from point (C) to the concentrated line weight in line segment i.
E_1, E_2	Error functions in the catenary formulation.
h	Vertical distance between support point (A) and support point (B).
H	Horizontal component of tension.
H_i	Horizontal component of tension in line segment i.
L	Horizontal distance between support point (A) and support point (B).
m	Parameter in the catenary formulation.
m_i	Parameter for line segment i.
M_i	Moment associated with line segment i.
R_i	Weight of line segment i.
s_i	Length of line segment i.
T	Tension in a line segment.
T_i^j	Tension at point j in line segment i.
T_{all}	Allowable tension in the skyline at point (A).
V	Vertical component of tension.
V_i^j	Vertical component of tension at point j in line segment i.
w	Weight per unit length for a line segment.
w_i	Weight per unit length for line segment i.
W_c	Weight of carriage.
W_L	Weight of log, assumed to be hanging vertically.
x	Horizontal coordinate in the catenary formulation.
x_i^j	Horizontal coordinate at point j in line segment i.
y	Vertical coordinate in the catenary formulation.
y_i^j	Vertical coordinate at point j in line segment i.
Δy	Vertical distance between support point (A) and the carriage position (C).

CONTENTS

1.0 INTRODUCTION

A logging skyline, used extensively for harvesting timber in rough terrain where a dense road system is undesirable, consists of cables or wire ropes suitably tensioned and anchored. A concentrated log load is supported in the span between the anchor points and transported to another point in the span (landing).

The designer of such a skyline must determine the load-carrying capability in order to establish mechanical and economic feasibility. Unfortunately, the direct computation of skyline load-carrying capability is not a simple procedure. The mathematical model of the skyline cable arrangement consists of a set of hyperbolic functions. As these hyperbolic functions are transcendental (that is, they cannot be manipulated algebraically for direct solution), it is difficult to obtain numerical solutions. An iterative (trial and error) procedure is required to determine numerical results. In terms of man-hours or computer time, these trial-and-error procedures can become very costly. This paper presents an iterative procedure, developed to provide numerical solutions to skyline problems, which has proven to be more efficient than other methods in the use of computer time.

1.1 Background

The mathematical formulation of the skyline problem is treated under the more general classification of catenary problems (Inglis 1963). Specific treatment of logging skylines supporting concentrated loads appeared in papers by Anderson (1921) and Mills (1932). These papers establish the mathematical formulation of the problem and serve primarily to point out the difficulties in obtaining numerical answers to skyline problems.

More recent works have transformed these formulations into tables and graphs for easier solution of skyline problems. The latest of these, by Lysons and Mann (1967), uses a graphical technique to establish the geometric characteristics of the skyline setting, together with tabulated information to determine the payload capability. This procedure can be used for single-span and multispan standing skylines with clamping or nonclamping carriages. The resulting payload capability is for the critical midspan position where capability is a minimum. Mann (1969) treats the problem of determining payload capability of running skylines in a similar manner. However, these methods are time-consuming and require many man-hours when a large number of roads is involved.

To avoid this time-consuming procedure, the designer can use a digital computer to directly compute the load-carrying capability. These direct computations will encounter the numerical complications discussed in the introduction. The more common iterative techniques, purely mathematical, are not concerned with the physical characteristics of the problem and, without artificial constraints, can diverge from the real answer or converge to answers which have no physical significance. The physical constraints must be introduced to control the iterative procedure of these standard numerical techniques. An example of this was provided by Suddarth's discussion (1970) of a direct computation of the load-carrying capability of the running skyline. His method was convergent, and numerical results for load-carrying capability were obtained. However, the complications associated with introducing the physical constraints confused the logical flow of the algorithm and caused it to be relatively slow to converge.

Our method provides a more efficient iterative procedure. The standard numerical schemes are not used; we have used a method which contains the physical

constraints as part of the procedure. This method is always convergent to the correct numerical result and generally requires not more than two or three iterations to converge. This method has been satisfactorily applied to both the standing and the running skylines and is also adaptable to analysis of other skyline configurations.

2.0 PROBLEM DESCRIPTION

Consider a single-span skyline with the geometry as shown in figure 1. In this cable system, the skyline is assumed firmly anchored at the tailhold while the line lengths are manipulated on drums at the yarder. Carriage vertical position is adjusted with the skyline length, while the horizontal location is controlled with the snubbing line. The carriage, shown in its simplest form in figure 2, is designed to run free on the skyline while held in position by the snubbing line. The problem is to determine the load-carrying capability at the carriage with the geometry shown and with a maximum operating tension specified in the skyline at the headspar.

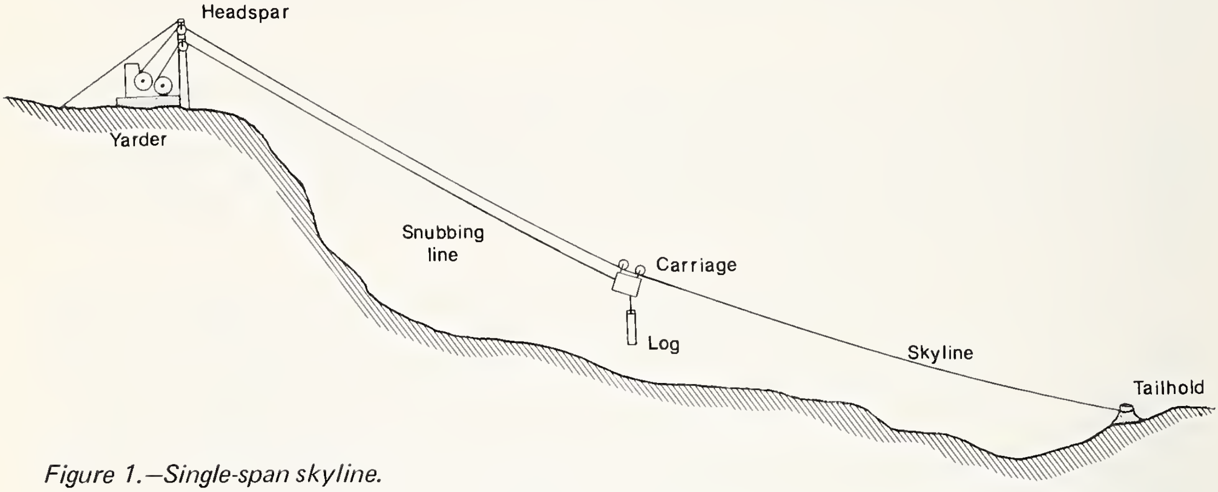


Figure 1.—Single-span skyline.

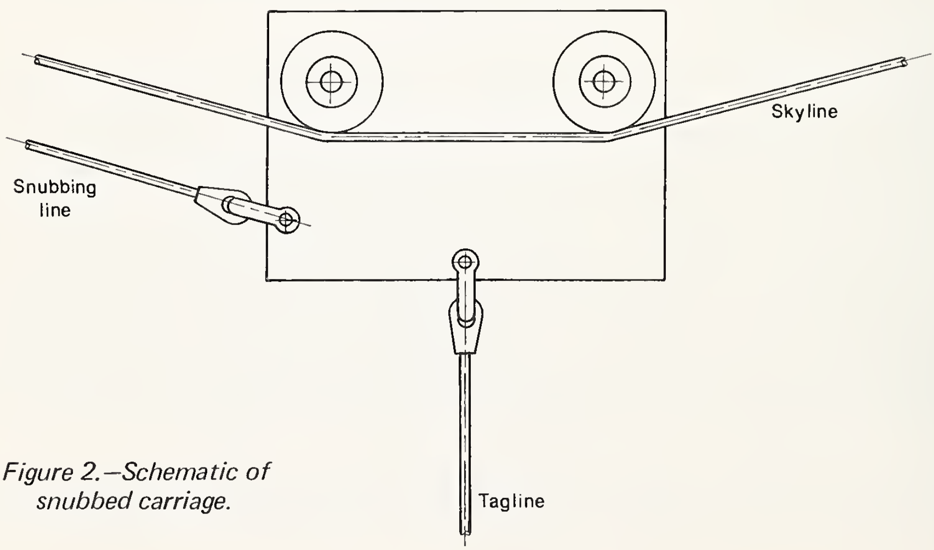


Figure 2.—Schematic of snubbed carriage.

3.0 CATENARY FORMULATION

The catenary formulation of the problem assumes completely flexible lines, with the cable weight distributed uniformly along the line segments (Inglis 1963). To develop the catenary expressions, it is convenient to adopt a reference coordinate frame, as shown in figure 3. In this coordinate frame, the governing expressions for each line segment become

$$y = m \cosh \frac{x}{m}, \tag{3.0.1}$$

$$T = wm \cosh \frac{x}{m}, \tag{3.0.2}$$

$$V = wm \sinh \frac{x}{m}, \text{ and} \tag{3.0.3}$$

$$H = wm. \tag{3.0.4}$$

The important parameter in these expressions is m. Once m has been determined, the problem has been solved and all other numerical values follow directly.

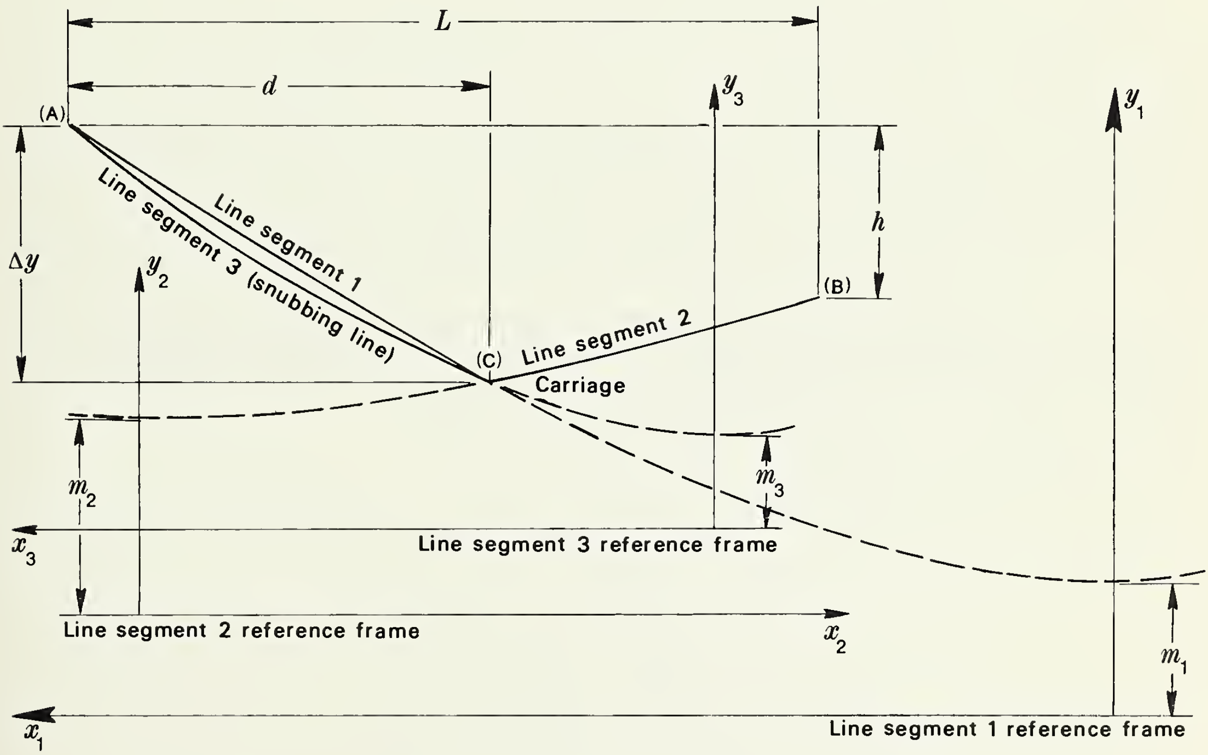


Figure 3.—Reference frames for the catenary formulation.

3.1 Boundary Conditions--Geometric

In the coordinate frames of figure 3, the known geometry of the problem can be introduced through six boundary conditions--two for each distinct line segment. These are:

Line segment 1 of skyline

$$y_1^a - y_1^c = \Delta y \qquad (3.1.1)$$

$$x_1^a - x_1^c = d \qquad (3.1.2)$$

Line segment 2 of skyline

$$y_2^b - y_2^c = \Delta y - h \qquad (3.1.3)$$

$$x_2^b - x_2^c = L - d \qquad (3.1.4)$$

Line segment 3 (snubbing line)

$$y_3^a - y_3^c = \Delta y \qquad (3.1.5)$$

$$x_3^a - x_3^c = d \qquad (3.1.6)$$

It is convenient to immediately incorporate these boundary conditions into the governing geometric equation (3.0.1). These can be combined to generate expressions for $\frac{x}{m}$, which are required arguments in the hyperbolic functions. These become:

Line segment 1 of skyline

$$x_1^a/m_1 = f_1(\Delta y, m_1, d) \qquad (3.1.7)$$

$$x_1^c/m_1 = g_1(\Delta y, m_1, d) \qquad (3.1.8)$$

Line segment 2 of skyline

$$x_2^b/m_2 = f_1(\Delta y - h, m_2, L-d) \qquad (3.1.9)$$

$$x_2^c/m_2 = g_1(\Delta y - h, m_2, L-d) \qquad (3.1.10)$$

Snubbing line

$$x_3^a/m_3 = f_1(\Delta y, m_3, d) \tag{3.1.11}$$

$$x_3^c/m_3 = g_1(\Delta y, m_3, d) \tag{3.1.12}$$

where these functions are defined as

$$f_1(p,q,r) = sinh^{-1}\left[\frac{p}{2q\ sinh\ \frac{r}{2q}}\right] + \frac{r}{2q}$$

$$g_1(p,q,r) = sinh^{-1}\left[\frac{p}{2q\ sinh\ \frac{r}{2q}}\right] - \frac{r}{2q}$$

3.2 Boundary Conditions--Force

In addition to horizontal and vertical forces balancing at the carriage, two other conditions must be met by the cable forces. The tension must be given at point (A) in the skyline, and the tension in segment 1 at (C) must be equal to the tension in segment 2 at (C). This latter condition is derived from the fact that the skyline goes under sheaves in the carriage which does not alter the tension. The boundary conditions can be expressed as follows:

Carriage force balances

$$w_1 m_1 sinh\ x_1^c/m_1 + w_2 m_2 sinh\ x_2^c/m_2 + w_3 m_3 sinh\ x_3^c/m_3 = W_c + W_L \tag{3.2.1}$$

$$w_1 m_1 + w_3 m_3 = w_2 m_2 \tag{3.2.2}$$

Line tension relationships

$$w_1 m_1\ cosh\ x_1^c/m_1 = w_2 m_2\ cosh\ x_2^c/m_2 \tag{3.2.3}$$

$$T_1^c = T_2^c = T_{all} - w_1 \Delta y \tag{3.2.4}$$

These line tension relationships (3.2.3) and (3.2.4) can be rearranged to the forms

$$E_1 = T_{all} - w_1 \Delta y - w_1 m_1 \cosh x_1^c / m_1 \tag{3.2.5}$$

and

$$E_2 = T_{all} - w_1 \Delta y - w_2 m_2 \cosh x_2^c / m_2 \tag{3.2.6}$$

If the values of the errors E_1 and E_2 are zero, or at least within an acceptable range of zero, the parameters of m_1 and m_2 represent a solution to the problem.

All the relationships necessary to obtain a catenary solution are now available. There are 10 unknowns (x_1^a, x_1^c, x_2^b, x_2^c, x_3^a, x_3^c, m_1, m_2, m_3, and W_L) and 10 independent equations [(3.1.7) through (3.1.12), (3.2.5), (3.2.6), and boundary conditions (3.2.1) and (3.2.2)]. As these relationships are transcendental, an iterative scheme is required to generate a solution.

4.0 FORCE BALANCE FORMULATION

A set of algebraic equations can be developed from force and moment balances on the line segments and carriage (fig. 4). For example, line segment 1 can be described by the expressions

$$V_1^a = R_1 + V_1^c \tag{4.0.1}$$

$$H_1 = \text{constant in the line segment} \tag{4.0.2}$$

and

$$V_1^a = R_1 \frac{e_1}{d} + H_1 \frac{\Delta y}{d} \tag{4.0.3}$$

Line segments 2 and 3 can be described by similar equations. As in the catenary formulation, the carriage force balances and line tension relationships provide four expressions. These equations are written in the nomenclature of this formulation as:

Carriage force balances

$$V_1^c + V_2^c + V_3^c = W_c + W_L \tag{4.0.4}$$

and

$$H_1 + H_3 = H_2 \tag{4.0.5}$$

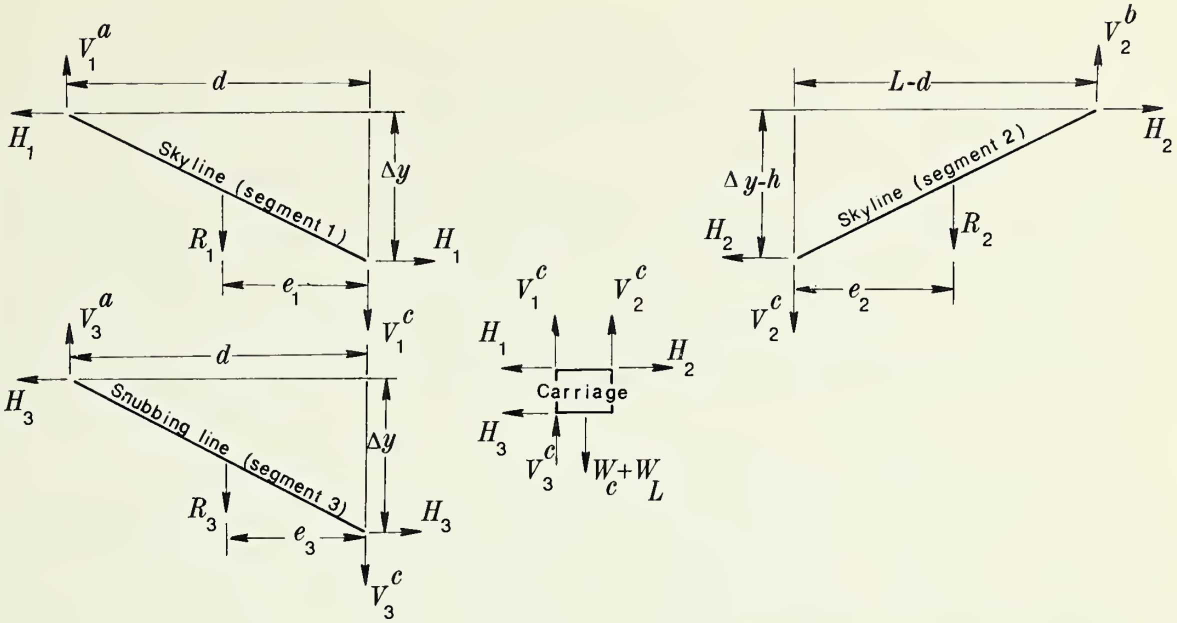


Figure 4.—Force balance formulation.

Line tension relationships

$$(V_1^c)^2 + (H_1)^2 = (V_2^c)^2 + (H_2)^2 \qquad (4.0.6)$$

and

$$T_1^c = T_2^c = T_{all} - w_1 \Delta y \qquad (4.0.7)$$

At this point, if the vertical forces, horizontal forces, and the pay-load weight W_L are considered unknowns, there are a total of 10 unknowns. Ten independent equations are available, and algebraic manipulation provides expressions which determine the horizontal forces directly. These expressions, listed in the order for solution with the necessary intermediate values, are

$$H_1 = \frac{-R_1 t_1 (t_2 -1) + [(R_1 t_1 (t_2 -1))^2 - (1+t_1^2)((R_1 (t_2 -1))^2 - (T_1^c)^2)]^{1/2}}{(1+t_1^2)}, \qquad (4.0.8)$$

$$H_2 = \frac{-R_2 t_3 (t_4 -1) + [(R_2 t_3 (t_4 -1))^2 - (1+t_3^2)((R_2 (t_4 -1))^2 - (T_2^c)^2)]^{1/2}}{(1+t_3^2)}, \qquad (4.0.9)$$

and

$$H_3 = H_2 - H_1 \qquad (4.0.10)$$

where

$$t_1 = \Delta y/d$$

$$t_2 = e_1/d$$

$$t_3 = \Delta y\text{-}h/L\text{-}d$$

and

$$t_4 = e_2/L\text{-}d \qquad (4.0.11)$$

Now we have an explicit solution for the horizontal tensions and, thus, the catenary parameters m_1, m_2, and m_3.

As values of horizontal tensions derived from these equations depend upon values for line weights and moment arms, the solutions for m_1, m_2, and m_3 will vary with the shape the lines adopt between these anchor points. If the lines were straight, rigid members, pinned at (A), (B), and (C), the weights and moment arms could be expressed as functions of the known quantities, namely,

$$R_1 = w_1((d)^2 + (\Delta y)^2)^{1/2} \qquad (4.0.12)$$

$$e_1 = d/2 \qquad (4.0.13)$$

$$R_2 = w_2((L-d)^2 + (\Delta y-h)^2)^{1/2} \qquad (4.0.14)$$

and

$$e_2 = (L-d)/2 \qquad (4.0.15)$$

and the horizontal tensions could then be computed. However, the lines are known to hang as catenaries, and their shapes are not known until the catenary solution is obtained.

In other words, we have arrived at a point similar to that reached in the catenary formulation. We have the equations available for computation of correct values of tension; however, they depend upon a knowledge of catenary line shapes which, in turn, depend upon the tension. Therefore, an iterative solution is also required in this formulation.

5.0 ITERATIVE TECHNIQUE

The catenary and force balance formulations, derived in the preceding sections, are both limited by difficulties which prevent solutions. The force balance formulation cannot provide accurate results without prior knowledge of the line weights and moment arms of the catenary line segments. The catenary formulation provides a complete set of equations; however, these equations are transcendental, and an iterative routine is required for solution. Convergence using the more common methods is slow, and the increase in complexity introduced by more cables in the system makes these direct mathematical methods unattractive. Our method is relatively insensitive to the complication of the cable system, and convergence is guaranteed by incorporating more of the physics of the system. In general, the method is an iterative procedure which uses the algebraic features of the force balance formulation and the tension and geometric expressions of the catenary formulation.

Direct solution by the force balance formulation is prevented by lack of knowledge concerning weights and moment arms of the lines hanging in catenary shapes. Weights and moment arms, however, could be computed if good approximations were available for the catenary parameters. These approximations can be provided by the force balance expressions. The line weights and moment arms are initially computed by assuming the line segments are straight, rigid, pinned members with the same unit weight as the actual cables. Once these approximations have been made, new weights and moment arms can be computed with the catenary expressions (see following section), and a second computation with the force balance equations will improve the approximations.

As the approximations of horizontal tension, and thus catenary parameters, improve, the approximations of line weights and moment arms improve, which improve the tension estimate, etc., until the procedure converges on the desired solution. This algorithm is discussed after the next section for the specific problem studied in this paper.

6.0 CATENARY LINE WEIGHTS AND MOMENT ARMS

To implement the iterative procedure described in the previous section, the hyperbolic function expressions for catenary line weights and moment arms must be available. These are derived below.

Consider the catenary line segment 1, as shown in figure 5. The line length of this segment can be expressed in terms of the appropriate catenary parameter m_1 and the values of x_1^a and x_1^c as

$$s_1 = m_1(\sinh x_1^a/m_1 - \sinh x_1^c/m_1) \qquad (6.0.1)$$

and the uniform line weight gives a concentrated load of

$$R_1 = w_1 s_1 \qquad (6.0.2)$$

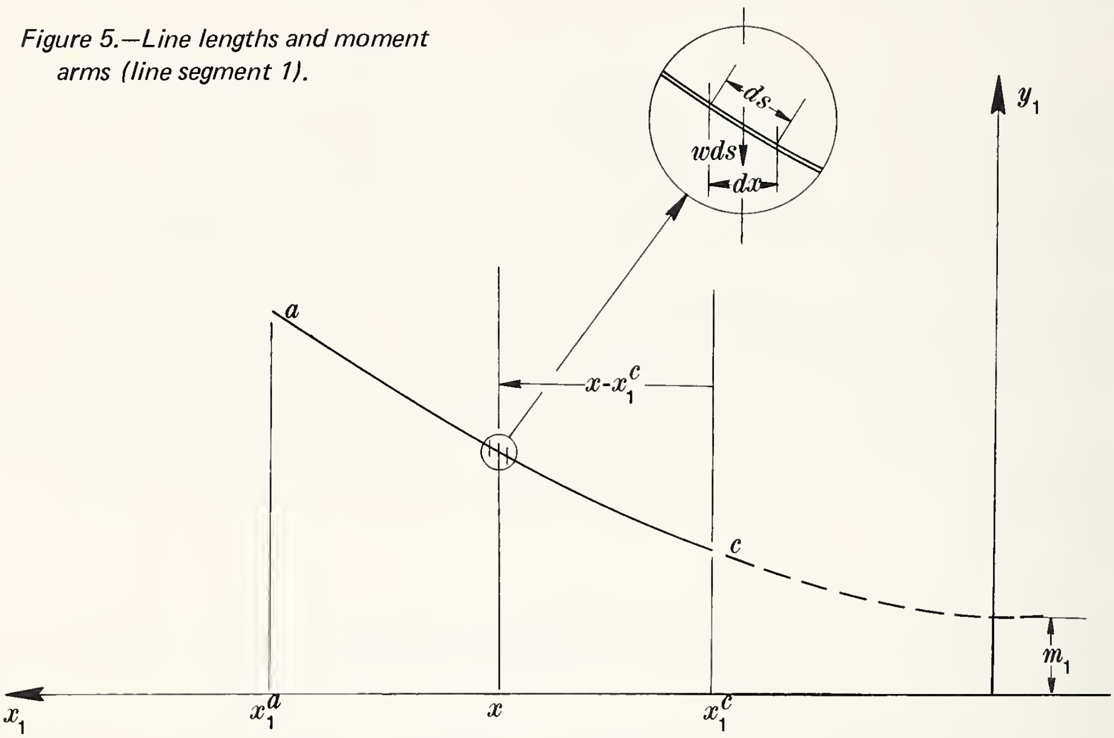
The moment arm expression is somewhat more involved. For a differential element, the moment about point (C) can be expressed as

$$dM_1 = (x - x_1^c)\, w_1 \cosh x/m_1 \, dx \qquad (6.0.3)$$

or, expressed in terms of the catenary formulation,

$$dM_1 = m_1 \left(\frac{x}{m_1} - \frac{x_1^c}{m_1}\right) w_1 \cosh \frac{x}{m_1} d\left(\frac{x}{m_1}\right) \qquad (6.0.4)$$

Figure 5.—Line lengths and moment arms (line segment 1).



The total moment about (C) for the entire line segment between (C) and (A) would be

$$M_1 = \int_c^a dM_1 = w_1 (m_1)^2 \int_{x_1^c/m_1}^{x_1^a/m_1} \left(\frac{x}{m_1} - \frac{x_1^c}{m_1}\right) \cosh \frac{x}{m_1} \, d\left(\frac{x}{m_1}\right) \qquad (6.0.5)$$

This yields a moment arm expression for line segment 1 of

$$e_1 = \frac{M_1}{R_1} = w_1 m_1 \; f_2 \, (x_1^a, \, m_1, \, x_1^c)/R_1 \qquad (6.0.6)$$

where

$$R_1 = w_1 g_2 \; (x_1^a, \, m_1, \, x_1^c) \qquad (6.0.7)$$

and

$$f_2 \, (p,q,r) = q \; \left[\left(\frac{p-r}{q}\right) \, sinh \, \frac{p}{q} - cosh \, \frac{p}{q} + cosh \, \frac{r}{q} \right]$$

$$g_2 \, (p,q,r) = q(sinh \, \frac{p}{q} - sinh \, \frac{r}{q})$$

An analysis of line segments 2 and 3 will give the similar results as shown below:

$$e_2 = w_2 m_2 f_2 \, (x_2^b, \, m_2, x_2^c)/R_2 \qquad (6.0.8)$$

$$R_2 = w_2 g_2 \, (x_2^b, \, m_2 \, , \, x_2^c) \qquad (6.0.9)$$

$$e_3 = w_3 m_3 f_2 \, (x_3^a, \, m_3, \, x_3^c)/R_3 \qquad (6.0.10)$$

and

$$R_3 = w_3 g_2 \, (x_3^a, \, m_3, \, x_3^c) \qquad (6.0.11)$$

7.0 ITERATIVE PROCEDURE FOR THE SINGLE-SPAN SKYLINE EXAMPLE

Consider the iterative procedure used for solution of the single-span skyline example. (For numerical example, see Appendix.) The subsections refer to blocks in the flow diagram (fig. 6), and the equations are those of this report.

7.1 Input

As formulated, this solution expects geometric quantities of L, d, Δy, and h, as well as skyline allowable tension, line weights per foot, and carriage weight as input.

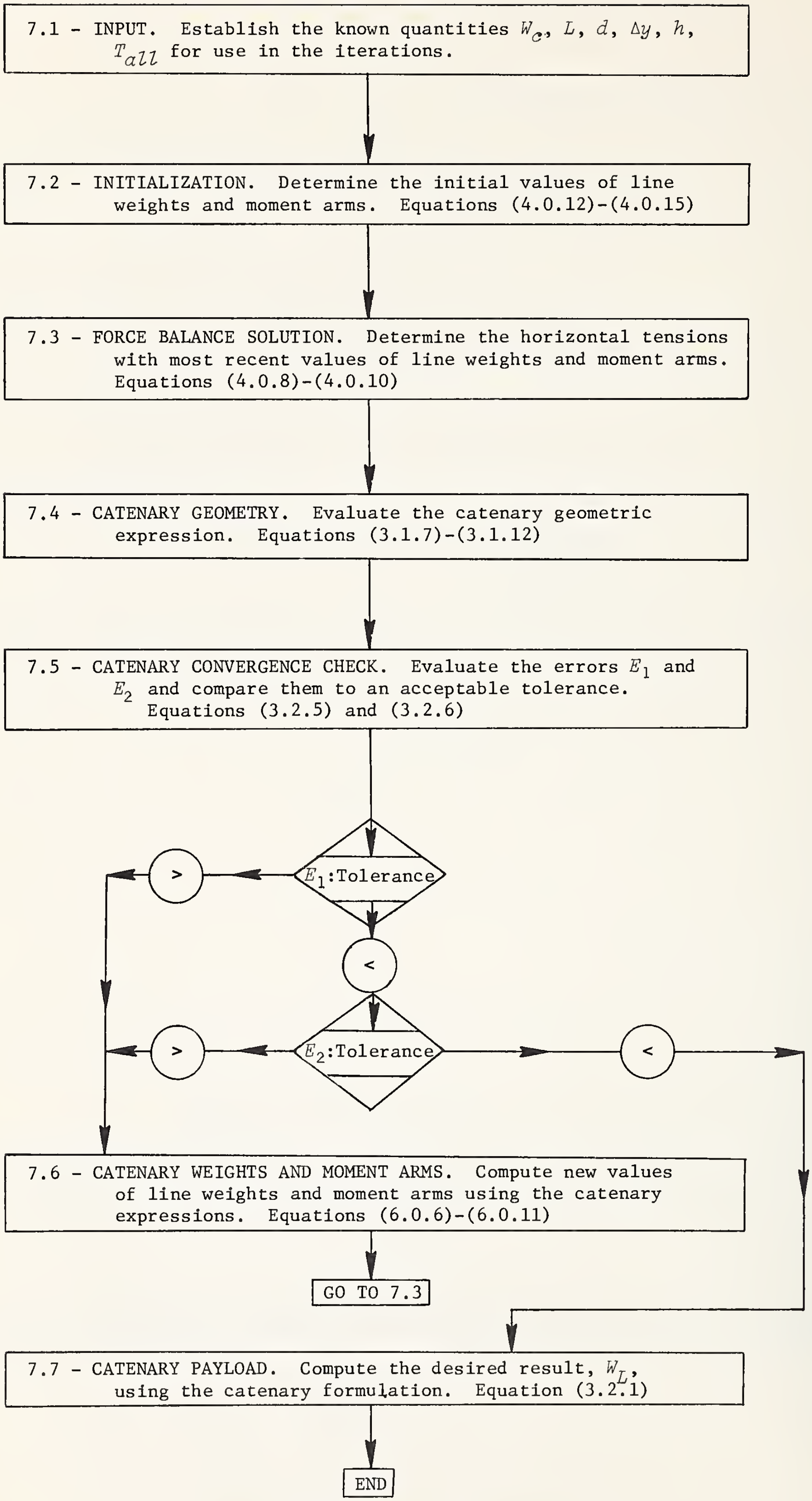


Figure 6.—Flow diagram of procedure.

7.2 Initialization

With the line segments approximated as straight, rigid members, pinned at points (A), (B), and (C), initial values for line weights and moment arms are computed.

7.3 Force Balance Solution

With the most recent values of line weights and moment arms, the force balance equations are solved for horizontal tensions which provide the catenary parameters.

7.4 Catenary Geometry

The catenary parameters generated in "Force Balance Solution" are used to compute the geometric functions expressed in equations (3.1.7) through (3.1.12).

7.5 Catenary Convergence Check

The catenary parameters and geometric functions are checked against the errors of equations (3.2.5) and (3.2.6) for convergence.

7.6 Catenary Weights and Moment Arms

The expressions of equations (6.0.6) through (6.0.11) are used to compute values for line weights and moment arms. The procedure returns to "Force Balance Solution" with these updated values.

7.7 Catenary Payload

The catenary expression for vertical force balance, equation (3.2.1), is used to compute payload W_L. Since the catenary parameters are available at this point, other tensions and components could be computed.

8.0 CONCLUSION

The algorithm presented in this paper has been extensively examined on a wide range of problems. In all cases, the convergence has been rapid, seldom exceeding two iterations. It should also be recognized that little additional complexity is introduced by adding lines or line segments to the cable system. Such cases will proportionately increase the number of force balance and catenary expressions involved; however, the logic of the procedure remains the same.

9.0 LITERATURE CITED

Anderson, Samuel H.
1921. Tensions in track cables and logging skylines. Univ. Wash. Eng. Exp. Sta. Bull. 13, 27 p., illus.

Inglis, Sir Charles Edward.
1963. Applied mechanics for engineers. Ch. IV, 404 p., illus. New York: Dover Publications, Inc.

Lysons, Hilton H., and Mann, Charles N.
1967. Skyline tension and deflection handbook. Pacific Northwest Forest and Range Exp. Sta. USDA Forest Serv. Res. Pap. PNW-39, 41 p., illus.

Mann, Charles N.
1969. Mechanics of running skylines. Pacific Northwest Forest and Range Exp. Sta. USDA Forest Serv. Res. Pap. PNW-75, 11 p., illus.

Mills, Russell.
1932. Tensions in logging skylines. Univ. Wash. Eng. Exp. Sta. Bull. 66, 30 p., illus.

Suddarth, Stanley K.
1970. Analysis of cable systems--the grapple rigged running skyline. Skyline Logging Symp. Proc., Oreg. State Univ.

APPENDIX

NUMERICAL EXAMPLE

To further clarify the algorithm discussed in this paper, a numerical example is presented. This example follows the procedure outlined in "Iterative Procedure for the Single-Span Skyline Example." Note that all calculations were done to 10 significant digits but have been rounded off for presentation.

7.1 Input

L = 2,500 feet
h = 1,000 feet
d = 700 feet
Δy = 600 feet

T_{all} = 64,000 pounds
$w_1 = w_2$ = 3.5 pounds per foot
w_3 = 1.85 pounds per foot
W_c = 5,000 pounds

7.2 Initialization

Equations used

$$R_1 = 3.5\ [(700)^2 + (600)^2]^{1/2} = 3{,}226.84 \qquad (4.0.12)$$

$$e_1 = 700/2 = 350 \qquad (4.0.13)$$

$$R_2 = 3.5\ [(1{,}800)^2 + (-400)^2]^{1/2} = 6{,}453.68 \qquad (4.0.14)$$

$$e_2 = 1{,}800/2 = 900 \qquad (4.0.15)$$

7.3 Force Balance Solution

$$H_1 = \frac{-(3226.84)(\frac{600}{700})(-0.5)+[(1382.9)^2-(1.7347)[(-1613.42)^2-(61900)^2]]^{1/2}}{1.7347} \qquad (4.0.8)$$

$$H_1 = 47786.0$$

$$H_2 = \frac{-(6453.48)(\frac{-400}{1800})(-0.5)+[(717.08)^2-(1.049)[(-3226.8)^2-(61900)^2]]^{1/2}}{1.049} \qquad (4.0.9)$$

$$H_2 = 59664.4$$

7.4 Catenary Geometry

$$m_1 = \frac{47786.0}{3.5} = 13653.1 \qquad (3.0.4)$$

$$m_2 = \frac{59664.4}{3.5} = 17047.0 \qquad (3.0.4)$$

$$\frac{x_1^a}{m_1} = sinh^{-1} \left[\frac{600}{2(13653.1)\ sinh \left[\frac{700}{2(13653.1)}\right]} \right] + \frac{700}{2(13653.1)} \qquad (3.1.7)$$

$$= 0.77660 + 0.02563 = 0.80223$$

$$\frac{x_1^c}{m_1} = 0.77660 - 0.02563 = 0.75096 \qquad (3.1.8)$$

$$\frac{x_2^b}{m_2} = sinh^{-1} \left[\frac{-400}{2(17047.0)\ sinh \left[\frac{1800}{2(17047.0)}\right]} \right] + \frac{1800}{2(17047.0)} \qquad (3.1.9)$$

$$= -0.22033 + 0.05280 = -0.16754$$

$$\frac{x_2^c}{m_2} = -0.22033 - 0.05280 = -0.27313 \qquad (3.1.10)$$

7.5 Catenary Convergence Check

$$E_1 = 61900 - (3.5)(13653.1)\ cosh\ (0.75096) = -5.640 \qquad (3.2.5)$$

$$E_2 = 61900 - (3.5)(17047.0)\ cosh\ (-0.27313) = -3.707 \qquad (3.2.6)$$

As these errors are greater than reasonable tolerance (say, 1.0 pound), the calculations will continue through an iteration.

7.6 Catenary Weights and Moment Arms

$$R_1 = 3.5(13653.1)[sinh\ (0.80223) - sinh\ (0.75096)] \qquad (6.0.7)$$

$$R_1 = 3227.0$$

$$e_1 = \frac{(3.5)(13653.1)^2}{3227.0} [(0.80223 - 0.75096) \sinh (0.80223)$$

$$- \cosh (0.80223) + \cosh (0.75096)] \qquad (6.0.6)$$

$$e_1 = 351.9$$

$$R_2 = 3.5(17047.0) [\sinh (-0.16754) - \sinh(-0.27313)] \qquad (6.0.9)$$

$$R_2 = 6456.5$$

$$e_2 = \frac{(3.5)(17047.0)^2}{6456.5} [(-0.16754 + 0.27313) \sinh(-0.16754)$$

$$-\cosh(-0.16754) + \cosh(-0.27313)] \qquad (6.0.8)$$

$$e_2 = 896.6$$

Now return to force balance with these new R's and e's.

7.3 Force Balance

$$H_1 = 47781.7 \qquad (4.0.8)$$

$$H_2 = 59660.8 \qquad (4.0.9)$$

7.4 Catenary Geometry

$$m_1 = 13651.9 \qquad (3.0.4)$$

$$m_2 = 17045.9$$

$$\frac{x_1^a}{m_1} = 0.77660 + 0.02563 = 0.80224 \qquad (3.1.7)$$

$$\frac{x_1^c}{m_1} = 0.77660 - 0.02563 = 0.75096 \qquad (3.1.8)$$

$$\frac{x_2^b}{m_2} = -0.220332 + 0.052799 = -0.167533 \qquad (3.1.9)$$

$$\frac{x_2^c}{m_2} = -0.220332 - 0.052799 = -0.27313 \qquad (3.1.10)$$

7.5 Catenary Convergence Check

$$E_1 = -0.000505 \qquad (3.2.5)$$

$$E_2 = -0.000245 \qquad (3.2.6)$$

These are acceptable errors. Proceed to next section.

7.7 Catenary Payload

$$m_3 = \frac{(3.5)(17045.9) - (3.5)(13651.9)}{1.85} \qquad (3.2.2)$$

$$m_3 = 6421.1$$

$$\frac{x_3^c}{m_3} = 0.776348 - 0.054508 = 0.72184 \qquad (3.1.12)$$

$$W_L = (3.5)(13651.9)\,sinh(0.75096)+(3.5)(17045.9)\,sinh(-0.27313)$$

$$+ (1.85)(6421.1)\,sinh(0.72184)-5000 \qquad (3.2.1)$$

$$W_L = 27191.8$$

This is the load-carrying capability of the equipment operating with the geometry input.

Carson, Ward W., and Mann, Charles N.
1970. A technique for the solution of skyline catenary equations. USDA Forest Serv. Res. Pap. PNW-110, 18 p., illus. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.

Presents an iterative technique for solution of skyline problems based on a catenary and force balance formulation. Trials of various skyline configurations show this technique is convergent in no more than three iterations. This information will be useful in developing a computer program for the analysis of suspended cable systems.

Carson, Ward W., and Mann, Charles N.
1970. A technique for the solution of skyline catenary equations. USDA Forest Serv. Res. Pap. PNW-110, 18 p., illus. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.

Presents an iterative technique for solution of skyline problems based on a catenary and force balance formulation. Trials of various skyline configurations show this technique is convergent in no more than three iterations. This information will be useful in developing a computer program for the analysis of suspended cable systems.

Carson, Ward W., and Mann, Charles N.
1970. A technique for the solution of skyline catenary equations. USDA Forest Serv. Res. Pap. PNW-110, 18 p., illus. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.

Presents an iterative technique for solution of skyline problems based on a catenary and force balance formulation. Trials of various skyline configurations show this technique is convergent in no more than three iterations. This information will be useful in developing a computer program for the analysis of suspended cable systems.

Carson, Ward W., and Mann, Charles N.
1970. A technique for the solution of skyline catenary equations. USDA Forest Serv. Res. Pap. PNW-110, 18 p., illus. Pacific Northwest Forest and Range Experiment Station, Portland, Oregon.

Presents an iterative technique for solution of skyline problems based on a catenary and force balance formulation. Trials of various skyline configurations show this technique is convergent in no more than three iterations. This information will be useful in developing a computer program for the analysis of suspended cable systems.

The mission of the PACIFIC NORTHWEST FOREST AND RANGE EXPERIMENT STATION is to provide the knowledge, technology, and alternatives for present and future protection, management, and use of forest, range, and related environments.

Within this overall mission, the Station conducts and stimulates research to facilitate and to accelerate progress toward the following goals:

1. Providing safe and efficient technology for inventory, protection, and use of resources.
2. Development and evaluation of alternative methods and levels of resource management.
3. Achievement of optimum sustained resource productivity consistent with maintaining a high quality forest environment.

The area of research encompasses Oregon, Washington, Alaska, and, in some cases, California, Hawaii, the Western States, and the Nation. Results of the research will be made available promptly. Project headquarters are at:

College, Alaska	Portland, Oregon
Juneau, Alaska	Roseburg, Oregon
Bend, Oregon	Olympia, Washington
Corvallis, Oregon	Seattle, Washington
La Grande, Oregon	Wenatchee, Washington

The FOREST SERVICE of the U.S. Department of Agriculture is dedicated to the principle of multiple use management of the Nation's forest resources for sustained yields of wood, water, forage, wildlife, and recreation. Through forestry research, cooperation with the States and private forest owners, and management of the National Forests and National Grasslands, it strives — as directed by Congress — to provide increasingly greater service to a growing Nation.